

M. RUFFINO

EXTENSIONS AS REPRESENTATIVE OBJECTS IN FREGE'S
LOGIC¹

ABSTRACT. Matthias Schirn has argued on a number of occasions against the interpretation of Frege's "objects of a quite special kind" (i.e., the objects referred to by names like 'the concept *F*') as extensions of concepts. According to Schirn, not only are these objects not extensions, but also the idea that 'the concept *F*' refers to objects leads to some conclusions that are counter-intuitive and incompatible with Frege's thought. In this paper, I challenge Schirn's conclusion: I want to try and argue that the assumption that 'the concept *F*' refers to the extension of *F* is entirely consistent with Frege's broader views on logic and language. I shall examine each of Schirn's main arguments and show that they do not support his claim.

In his essay "*Über Begriff und Gegenstand*" (from 1892), Frege famously addresses an objection raised by Beno Kerry against his sharp ontological distinction between concept and object. Kerry claims, against Frege, that the ontological status of something as a concept or as object is not absolute, that is to say, something that seems like a concept may "behave" like an object in certain contexts, and vice-versa. Kerry uses the famous example of the expression 'the concept *horse*' to illustrate his point. The thing designated by this expression seems to be a concept in contexts like 'Silver falls under the concept *horse*'. But in contexts like 'The concept *horse* is easy to grasp', 'the concept *horse*' turns out to designate an object due to its position as grammatical subject. Since there is no reason to suppose that 'the concept *horse*' refers to different things in each one of these contexts, Kerry concludes that the status as concept or as object of the thing designated is relative and not absolute, contrary to Frege's view.

An essential part of Frege's reply to Kerry's criticism is his well known claim that, due to the presence of the definite article in 'the concept *horse*' the expression has to refer (if it has a reference at all) to an object, and not to a concept. And this is so in all contexts. There is here, as Frege claims, a systematically misleading effect of ordinary language: it makes us refer to an object, while in fact we intend to refer to a concept. The expression 'the concept *horse*' refers, according to Frege, to an object that "represents" the intended concept in logical investigations (*KS* 170).¹ Hence Frege's apparently paradoxical dictum that "the concept *horse* is not a concept" (*KS* 170).



Frege's view on this issue is certainly not unproblematic, and I shall not provide a systematic defense of it in this paper. Indeed, I think that a lot could be said against his confidence that the presence of the definite article is a reliable sign that expressions behave as proper names and hence refer to objects.² But I think it important first to clearly understand what exactly Frege has in mind at this point. For his view on this issue is, as I believe, closely related to his broader view on logical objects developed in his writings up to *GGA II* (1903).

Albeit Frege explicitly claims that the reference of 'the concept *horse*' is an object and not a concept, he is less clear about which object it is supposed to be. It seems, however, that, if an expression of the form 'the concept *so-and-so*' has to refer to an object, the most natural candidate as reference would be the extension of the corresponding concept (which, for Frege, is an object). If this is so, the expressions 'the concept *horse*' and 'the extension of the concept *horse*' would have the same reference for Frege, namely, the extension of the concept *horse*. To the best of my knowledge, there is no point in Frege's writings where he explicitly and unequivocally states the identification above. (There is also no place where he denies it either.) But, as it seems to me, there is some good evidence that he implicitly accepted this identification, at least in his writings up to *GGA II* – i.e., at least before the discovery of Russell's paradox. This interpretation would help, for example, to explain Frege's remark in the footnote to *GLA* §68 about the possible substitution of 'concept' for 'extension of the concept' in his famous definition of cardinal numbers "the number that belongs to the concept *F* is the extension of the concept '*equinumerous with the concept F*'" (*GLA* §68). The remark seems at first sight to be at odds with Frege's own perspective, since extensions are objects in his view, and hence essentially distinct from concepts. But a crucial detail (as Frege stresses, e.g., in *KS* 172) is that in the definition of cardinal number, the expression 'concept *F*' is preceded by the definite article and hence refers to an object. If we accept that Frege holds the identification in question, then the remark above can be seen as perfectly consistent with his own views on concepts and objects. Moreover, as I argued elsewhere,³ if we accept that Frege tacitly accepted this identification before 1903, we can understand why he regarded extensions as having a special status as logical objects:⁴ they are, so to speak, "objectified" concepts, and since logic deals primarily with concepts, extensions are the most primitive logical objects. Therefore, we can understand why he thought that the reduction of numbers to extensions is absolutely essential for his logicist project. We can also better understand his choice of Axiom V (instead of Hume's principle) as a fundamental law of logic,⁵ and his reluctance in giving up

Axiom V even after the discovery of Russell's paradox: Axiom V is a translation in Frege's own terminology (i.e., the terminology of extensions and value-ranges) of the law "the concept F is identical with the concept G if and only if they have the same value for all objects as arguments." This later law is, as Frege repeatedly remarks,⁶ the analogue of Leibniz's law of identity for concepts – and, hence, it is as fundamental and indispensable as Leibniz's law.

The point of this paper, however, is not to argue in detail for this interpretation. It is, rather, to examine the position of a Fregean scholar who argues against it. In his paper "*Begriff und Begriffsumfang. Zu Freges Anzahldefinition in den Grundlagen der Arithmetik*" (1983), Matthias Schirn argues against attributing to Frege the identification of the reference of 'the concept F ' with the reference of 'the extension of the concept F '. Later, the main points of this paper regarding the identification were repeated and further developed in "Frege's Objects of a Quite Special Kind" (1990). Finally, his main points show up again in his more recent "On Frege's Introduction of Cardinal Numbers as logical Objects" (1996). Now, because Schirn was the first to argue in detail⁷ against the interpretation that I favor – i.e., that Frege accepts the identification – and because the thesis he is arguing against is, as I indicated above, essential for the correct understanding of Frege's notion of logical objects, it is worth examining Schirn's main arguments in detail and seeing where they go wrong. This is what I shall do in this paper. My purpose is, hence, mainly negative. I will concentrate on the first two sections of Schirn's second paper (1990) since it contains a good synopsis of the main arguments used in all of the three papers. I shall argue that each one of Schirn's main arguments is essentially flawed – they are insufficient to rule out the interpretation that, at least before 1903, Frege contemplated the identification thesis.

The title of Schirn's second paper comes from Frege's remark in "*Über Begriff und Gegenstand*" that in an expression like 'the concept F is realized', what is said to be realized is an "object of a quite special kind" (KS 174).⁸ Schirn seems to take this description as a sign that the intended objects are, in Frege's view, of some mysterious or problematic nature. This places, from the beginning, Frege's introduction of these objects in an unfavorable light, and makes it appear like a desperate ontological inflationism, the sole purpose of which is to save Frege's distinction between concepts and objects from Kerry's charge. As I shall argue later, there is another (more adequate) way of understanding the description "of a quite special kind" that removes the flavor of obscurantism suggested by Schirn's interpretation.

There are basically four different questions addressed by Schirn regarding Frege's semantical views about the expression 'the concept F ':

- (i) Can we say that Frege was seriously committed to the semantical thesis that the expression 'the concept F ' is a singular term (and hence to the ontological claim that there are objects of a quite special kind)?
- (ii) In case the answer to (i) is affirmative, is Frege's thesis acceptable?
- (iii) Does 'the concept F ' refer to the same thing as 'the extension of the concept F ' in Frege's view?
- (iv) In case the answer to (iii) is affirmative, is Frege's thesis acceptable?

Schirn advocates a negative answer to all of them, although sometimes it is not quite clear which particular question is being addressed by each particular argument. Despite the fact that Frege explicitly holds the thesis mentioned in (i) in "*Über Begriff und Gegenstand*", Schirn mentions some textual evidence from the later writings to show that this commitment was not serious or soon realized to be incoherent. But we should keep in mind that Frege's position underwent some radical changes after the discovery of Russell's paradox in 1902.⁹

Most of Schirn's considerations in his article seem to be variations of the following three basic arguments¹⁰ that he presents on pp. 28–9:

1. If the expression 'the concept F ' is to be seen as having the same reference as 'the extension of the concept F ', then we could substitute 'the extension of the concept F ' for 'the concept F ' in the second expression. The resulting expression would be 'the extension of the extension of the concept F ', and it must have the same reference as the original one, since the reference of a complex expression is a function only of the reference of its constitutive parts. Schirn presents two reasons for considering this conclusion absurd:

1.1. The portion 'the extension of _____' of the original expression must be the name of a second-order function because it takes concepts as arguments. But 'the extension of the concept F ' is the name of an object, so that the resulting expression cannot be well-formed (1990, 28).

1.2. If the resulting expression 'the extension of the extension of the concept F ' has the same reference as 'the extension of the concept F ', then we could in principle reiterate the substitution of 'the extension of the concept F ' for 'the concept F '. So that 'the extension of the extension of the extension of the concept F ' would have the same reference as 'the extension of the concept F ', and so on *ad infinitum*. Because of this regress, Schirn concludes

Obviously, no one would seriously assume that in Frege's view "the extension of the extension of the extension of the concept F " designates the same object as does "the concept F ". (ibid.)

2.1. If Frege accepted seriously the identification between the reference of 'the concept F ' and 'the extension of the concept F ', then it would be a mystery why he never clearly stated it (ibid., 29).

2.2. If Frege had considered seriously the identification, then he would be committed, according to Schirn, to an awkward thesis, namely that

[...] the assertion that something is realized – as the word is being understood in a sentence like "The concept *horse* is realized" – can be truly made only about extensions of concepts as objects of a quite special kind. But he is most unlikely to have held such a strange view. (ibid.)

3. There are failures in the analogy between extensions and objects of a quite special kind that undermine the identification in question. One of the most important differences between the two kinds of objects is the justification for their recognition: the existence of objects of a quite special kind is, according to Schirn, "justified only by appeal to a specific awkwardness of natural language" (p. 29). The situation is completely different in Frege's recognition of extensions. The existence of extensions is not problematic in Frege's eyes, and their introduction is necessary to carry out the logicist definition of numbers on a methodologically safe basis (ibid.). The different justifications for the two kinds of objects speaks, in Schirn's view, against the identification of them.

As I see it, argument 1.1 primarily addresses questions (i) and (ii) above; argument 1.2 primarily addresses question (iii); arguments 2.1 and 2.2 are also directed to question (iii); and argument 3 is directed to questions (iii) and (iv). I will first examine arguments 2 and 3, and only then turn to argument 1. As I will argue, arguments 2.2 and 3 are weak. Argument 2.1 is only partially justified, but not decisive at all. Only argument 1 presents some problems that the identification may face, but it does not count as significant evidence against it. It can, at most, show that Frege did not fully spell out the consequences of the views expressed in "*Über Begriff und Gegenstand*".

The worry expressed in 2.1 is partially justified. Indeed, it is intriguing why Frege never explicitly made the identification in question. But it is not decisive, since Frege never denied the identification before 1903. A careful analysis of the relevant textual evidence from Frege's writings before 1903 suggests rather the opposite question: if Frege saw any ontological divergence between the reference of 'the concept F ' and 'the extension of the concept F ' in his writings before 1903, then why did he not make the difference explicit? Such a lacuna would be especially mysterious since he brought the two expressions so close together¹¹ and formulated exactly the same identity-conditions for both kinds of objects on a number of occasions.¹² One of the reasons that may have deterred Frege from stress-

ing the identity is the propensity that the term ‘the concept *F*’ has to cause misunderstanding, since it refers to something different than what we originally intend.¹³ (Later in his writings he will recommend the avoidance of expressions of this form in logic.) The identification can easily lead to the false impression that concepts are to be identified with their extensions. But the sharp distinction between concept and object is too precious for Frege, and it is perfectly imaginable that he would have avoided any way of expressing himself that could induce the reader to lose sight of the distinction. But this is highly speculative; here only speculations are possible.

I turn now to Schirn’s point in 2.2. It is not clear why he thinks that the consideration of extensions as objects of special kind would be at odds with any of Frege’s views. No textual evidence is provided for Schirn’s claim that I quoted. But I suppose that Schirn is contrasting extensions, the status and properties of which are relatively clear for Frege, with objects of a special kind, the status and properties of which are (supposedly) obscure. It all seems to depend, however, on what is meant by the qualification “of a quite special kind”. As I pointed out at the beginning, Schirn seems to regard this description as somehow pejorative. He seems to understand “objects of a quite special kind” as almost synonymous with “mysterious objects” (e.g., on p. 39). It is possible, however, to differently understand what Frege means by this description. A plausible suggestion is that “of a quite special kind” means simply that the objects in question are connected in a special way with concepts. First, they can only exist if concepts are already given (*KS* 209–10, 225; *NS* 199; *WB* 122). Second, they are objects that are very close to concepts at the level of reference (“*sehr enge mit ihm [the concept] zusammenhängt*” (*NS* 134)), although they are categorically distinct from the latter. Extensions have these properties, as Frege explicitly says in (“*Ausführungen Über Sinn und Bedeutung*”) (*NS* 134), while objects of no other type seem to have it. Finally, these objects (extensions) are of a special kind because, as representative of concepts, they play a central role in logical investigations (*KS* 171, 199). In “*Booles rechnende Logik und die Begriffsschrift*”, when talking about how concepts definable in his system are fruitful (in comparison with the concepts definable in Boole’s logical calculus), Frege switches the discourse to extensions without a warning, and this suggests that he was taking extensions to be representative of concepts (*NS* 37–8). (Otherwise he would be falling into the same kind of confusion between a concept and its extension that he repeatedly criticizes in other philosophers.) Thus, if extensions are what Frege means by objects of a quite special kind, this makes them not the least obscure. But independently of the way we understand “of a quite

argument seems to be misconceived, for if the identification thesis is correct (and this is what his argument tries to refute by a kind of *reductio*), then these objects of a quite special kind are simply extensions, and hence there is nothing mysterious about them in Frege's view after all – at least not before 1903.

Schirn's argument 3 seems weak to me. First, it is not exactly correct to say that the existence of objects of a quite special kind is *justified* by appeal to a specific awkwardness of natural language. What is justified by the awkwardness of language is the fact that we refer to an object when we actually want to refer to a concept by means of an expression like 'the concept *F*'. These objects of a special kind are not introduced to account for a strange peculiarity of language; it is this peculiarity of language that forces us to refer to these objects in some contexts. But even if Schirn's claim of disanalogy were correct, it is not at all incompatible with the claim that the two kinds of objects are actually the same. The asymmetry that Schirn mentions between Frege's attitude towards the expression 'the concept *F*' and his attitude towards 'the extension of the concept *F*' in no way precludes the possibility that they have the same reference. In the same way that any difference in our attitude regarding numbers and our attitude regarding extensions would not prevent the former from being identical with some of the latter, for being an extension is, according to Frege, the true essence of a number. Whether the object designated by 'the concept *F*' is related in a particular way to the extension of the concept *F* or not is an objective matter which is completely independent of the processes that lead us to recognize extensions or objects of a special kind as existents.

Argument 1 is stronger. It points out some inconveniences that arise for the identification in question. Throughout Schirn's paper, a multiplicity of minor arguments are presented that can be seen as variations of 1.2 and (especially) of 1.1. They all indicate how sentences that involve the expression 'the concept *F*' or 'the extension of the concept *F*' and that are taken to be true in the informal language turn out to be (apparently) meaningless if both expressions are seen as interchangeable. The argument is stronger, but still not convincing. It points out some consequences of Frege's semantical theses in "*Über Begriff und Gegenstand*" that may possibly have escaped his notice. But once it is recognized that there may be a disparity between what is said and what is meant in ordinary language discourse about concepts, then it is natural to expect that the disparity should proliferate to many other contexts, and not simply be present in the particular cases mentioned by Frege "*Über Begriff und Gegenstand*". That is to say, the gap between what is meant and what is said should be expected in many other statements formulated in the informal language.

This will become clearer in my discussion of the examples that Schirn uses to refute the identification thesis. I will first consider 1.1 and some related arguments. Then I will consider 1.2 in connection with Schirn's refutation of an argument for the identification thesis.

Let me start by recalling the example presented in argument 1.1. Schirn claims that the substitution of 'the extension of the concept F ' for 'the concept F ' in 'the extension of the concept F ' produces an expression with no reference because 'the extension of _____' is the name of a second-order concept, whereas 'the extension of the concept F ' is the name of an object. Now, the only view consistent with Frege's claim that 'the concept F ' is a singular term is the one that considers the portion 'the extension of _____' from 'the extension of the concept F ' as the name of a first-order function, and not of a second-order one (as Schirn seems to assume). If this is so, the substitution of 'the extension of the concept F ' for 'the concept F ' in the original expression must yield a referential name, no matter how odd this name may appear. Schirn briefly contemplates this possibility and dismisses it in footnote 5 on the basis of the following claim:

It is quite obvious, however, that in *GLA* Frege intends to use "the extension of ..." as a second-level or third-level function-name. (ibid., 55)

But this claim seems irrelevant in the context of the present discussion, since Frege's crucial point in "*Über Begriff und Gegenstand*" was exactly that there is an irreducible distance between what is meant and what is actually said in natural language, and it is in a natural language that *GLA* is formulated. The awkwardness of natural language that opens a gap between what is meant and what is said in the case of 'the concept F ' necessarily has a similar effect in the case of other expressions. One of these cases is 'the extension of _____', which is *meant to refer* to a second-order function, but which *actually refers* to a first-order one.

If one keeps in mind what is said in "*Über Begriff und Gegenstand*" and elsewhere about the inadequacy of natural language to express logical properties and relations, then one can see that what is said in the informal part of *GGA* frequently diverges considerably from what Frege meant (and what we as readers understand perfectly well as being his intention). What is said in the informal part must be seen as elucidatory remarks, preparatory to what will be expressed in a more appropriate way in the formal language.¹⁴

Besides the three basic arguments that I mentioned before, Schirn considers and rejects in the first section of his paper two possible arguments that could be raised in favor of the identification thesis. The first argument criticized by Schirn is the following: Frege could have considered the two

expressions 'the extension of the concept *square root of 4* is not empty' and 'the concept *square root of 4* is not empty' to be synonymous. This is so because he could recognize as synonymous not only the pair of sentences (I) 'there is at least one square root of 4' and (II) 'the extension of the concept *square root of 4* is not empty', but also the pair (III) 'the concept *square root of 4* is realized' and (IV) 'the concept *square root of 4* is not empty'. Since in "*Über Begriff und Gegenstand*" he explicitly recognizes (I) and (III) as synonymous, and since synonymy is a transitive relation, then he could also recognize (II) and (IV) as synonymous. In this case, not only 'the extension of *square root of 4*' and 'the concept *square root of 4*' have the same reference, but also the same sense (p. 29). Schirn's reasons for the rejection of this conclusion are two. The first is the following:

There is, however, every reason to believe that, in his [Frege's] view, the two expressions do not have the same reference, let alone the same sense. Frege obviously did not recognize that one could derive this untenable result from his exposition concerning the use of an expression like "the concept *square root of 4*" (1990, 30)

No evidence is provided for this claim. As it is stated, the claim seems rather to beg the question, and to presuppose the interpretation that Schirn is arguing for. So it is not really a reason at all. The second reason is the following:

[...] for Frege the function-name "the extension of _____", which forms a part of the term "the extension of the concept *square root of 4*", clearly has a sense and contributes to determining the sense of the more complex term. Both singular terms under consideration could express the same sense only if "the extension of _____" were semantically vacuous. (ibid.)

Actually we have no textual evidence for attributing to Frege the view that 'the extension of _____' is not semantically vacuous *in particular contexts like the one of this expression*. But even if we take for granted that Frege holds this view, I think that Schirn's conclusion about the semantical vacuousness of 'the extension of _____' in case the two expressions have the same sense does not follow. As I argued before, if 'the concept *F*' is a singular term in 'the extension of the concept *F*', then 'the extension of _____' must be the name of a first-order function. Therefore, the substitution of any singular term for 'the concept *F*' must yield a referential expression. But what could 'the extension of *a*' possibly refer to if '*a*' is not a name of the form 'the concept *F*' but instead a name like, e.g., 'Julius Caesar'? The simplest way of retaining consistency with Frege's original claim is to assume that the whole expression would refer simply to *a*. In the same way that 'the extension of the concept *F*' refers to the same thing as 'the concept *F*' (that is to say, to the extension of the concept *F*), 'the extension of *Julius Caesar*' has the same reference as '*Julius Caesar*'.

Thus ‘the extension of _____’ is a name of the identity-function, i.e., if we abbreviate by ‘ $E(x)$ ’ the expression ‘the extension of _____’, then ‘ $E(a) = a$ ’ for every singular term ‘ a ’. Obviously a consequence of this is that ‘ $E(E(E \dots E(a)) \dots) = E(a)$ ’ and ‘ $E(a) = a$ ’ are true statements. This does not imply, however, that ‘ $E(x)$ ’ is semantically vacuous; it is not so because it is a name of a function (identity), and hence has both a reference and a sense. I am not claiming that Frege actually formulates anything like this view for the expression ‘the extension of _____’. But this could be a way out for the problem that Schirn raises in this passage and it is entirely consistent with everything else that Frege says.¹⁵ The conclusion that I draw is that the expression ‘the extension of _____’ that occurs in ‘the extension of the concept *square root of 4*’ does not have necessarily to be considered as semantically vacuous if it is seen as having the same reference as ‘the concept *square root of 4*’, although it admittedly does not seem, at first sight, to add to the sense of the latter. This reasoning provides an indication of how one could deal with the difficulty pointed out in Schirn’s argument 1.2 as well.

The second argument for the identification opposed by Schirn is one that I briefly mentioned before: the criterion of identity for the objects of a quite special kind denoted by ‘the concept F ’ and ‘the concept G ’ is, according to Frege, the mutual subordination of F and G – i.e., the fact that they yield the same value for any object as argument (*NS* 131–2, 195–6; *GGA* II §147). But this is exactly the criterion of identity for the extension of F and G . This strongly suggests that both kinds of expressions must refer to the same kind of object. Against this argument, Schirn points out the following fact:

According to Frege, when we use the expression “the concept *horse*”, for instance, we intend to refer to a concept, but actually refer to an object, whereas, when we use the expression “the extension of the concept *horse*” we want to refer to an object and in fact do so. Thus, although both expressions count as singular terms, our attitude towards them as language-users involves a striking difference: the use of the second term, unlike the use of the first, does not give rise to an unsolvable antagonism between what we *mean* with our words and what we actually *say* with them. (1990, 31)

This argument seems congenial to argument 3 that I discussed before. And it is, in my view, equally weak. The fact that we have two different attitudes as speakers towards each one of the expressions in no way imply that their reference is different. The antagonism between what we mean and what we say when we employ the expression ‘the concept F ’ has nothing to do with what this expression refers to or with what it cannot refer to. The situation would be analogous to the case of two definite descriptions of the same object: our attitude as speakers towards each one of them may be, for some reason, entirely different. But this does not change the fact that

both descriptions do refer to the same object. It is a consequence of Frege's realist perspective that the existence of a relation between the reference of the expressions 'the concept F ' and of 'the extension of the concept F ' is independent of our attitude as speakers towards each expression.

The conclusion that I draw from these considerations is the following: leaving aside the historical doubts manifested by Schirn (which, as I indicated, are not quite justified), the systematic objections made against the identification in question do not constitute an insuperable problem for it. They just point out some apparently strange consequences of the identification. But there is nothing more strange about them than there is about Frege's claim that 'the concept F ' refers to an object. Now one could ask whether Frege had indeed good reasons for the latter claim, and I do not want to defend his view in this paper. Frege certainly owes us an explanation of why it is the case that sometimes – but not always – we end up referring to something different than the intended entities. But, at any rate, if he was ready to concede that by using expressions like 'the concept F ' we open a gap between what is meant and what is said, then there is no reason to suppose that he would reject the existence of a similar gap in the case of many other expressions of the informal part of his logic, including the examples adduced by Schirn.

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NOTES

¹ See references for the abbreviations of Frege's works used in this paper.

² In his later writings (1924/25) Frege refers back to his work previous to the discovery of Russell's paradox and comments that he himself was victim of the "illusion" ("*Täuschung*") created by the presence of the definite article in expressions like 'the extension of the concept *fixed stars*' and 'the concept *fixed star*'. According to Frege's later view, the definite article creates the appearance ("*Schein*") that the expressions refer to objects (*NS* 288–9).

³ 1996, chapter I.

⁴ This view is suggested, e.g., in *GGA* II 253; *WB* 121, 213. Frege seems to imply that extensions are not just a kind of logical object among others, but also the paradigmatic case of logical objects.

⁵ Frege's Axiom V is the law according to which two concepts (functions) F and G have the same extension (value-range) if and only if, for any object, F and G have the same value for this object as argument. Hume's principle is the law according to which two concepts F and G have the same number associated with them if and only if they are equinumerous, i.e., there is a bijection between the objects falling under F and the objects falling under G . Hume's principle was considered by Frege as a possible definition of cardinal numbers in *GLA* §§62–4, and then rejected due to the argument developed in *GLA* §§66–7.

⁶ E.g., in *NS* 131–2, *NS* 195–6, *GGA* II §47.

⁷ Of course I do not mean that Schirn was the only one to hold this view. Other qualified scholars held similar views before him. Sluga (1980, pp. 143–2) and Khatchadourian (1956, p. 94) think like Schirn, i.e., that Frege does not have extensions in mind as representative objects for concepts. But Schirn offered the most comprehensive defense of this interpretation so far. On the other hand, Bartlett (1961, p. 62), Burge (1984, especially p. 16 and p. 28) and Parsons (1984) defended the same view that I do. Thiel (1968, p. 67) conjectures that Frege found no decisive evidence for deciding one way or another and, consequently, consciously avoided giving a definite answer to this question. In my (1996) and (1999) I argue that, although there is no decisive evidence for it, the available (indirect) evidence is overwhelmingly in favor of the assumption that Frege tacitly accepted the identification in question.

⁸ The statement 'Julius Caesar is realized' is meaningful and false according to Frege, since 'Julius Caesar' is not the name of an object of a quite special kind, but of an ordinary object (*KS* 174).

⁹ For a detailed historical account of these changes, see Burge 1984.

¹⁰ I divided Schirn's arguments 1 and 2 into sub-arguments for the purpose of discussion. In Schirn's paper this division is not made.

¹¹ E.g., in *GLA* §68 and *KS* 112. In *NS* 116 (footnote) Frege explicitly says that he does not see any fundamental difference between both expressions.

¹² E.g., in *NS* 132–3, 195–6; *GGA* II §147.

¹³ In *NS* 116, in a passage from "*Über Begriff und Gegenstand*" that was suppressed in the published version of the text, Frege refers back to the note in *GLA* §68 in which he says that 'the concept' could replace 'the extension of the concept' in the definition of cardinal numbers. Frege explicitly says that he did not base anything on this remark in order not to have to deal with the misunderstanding that it could potentially cause.

¹⁴ The same reasoning above applies to two further counter-examples that Schirn presents against the identification thesis. The first one is: (*S*) 'The concept $x = x$ falls under the concept $(\forall x)\varphi(x)$ ' which is meaningful and true. But if we substitute 'the extension of the concept $x = x$ ' for 'the concept $x = x$ ' in (*S*), then we obtain (*S'*) 'The extension of $x = x$ falls under the concept $(\forall x)\varphi(x)$ ', which is meaningless according to Schirn, since $(\forall x)\varphi(x)$ is a second-order concept and extensions are objects (1990, 38). (This example appears to work not just against the identification thesis, but also against Frege's explicit claim (e.g. in *KS* 169) that 'the concept $x = x$ ' is a genuine singular term.) The second example is: (*Q*) 'The concept $x = x$ has, for every object as argument, the True as its value'. This statement is meaningful and true in Frege's system. But the substitution of 'the extension of the concept $x = x$ ' for 'the concept $x = x$ ' produces a meaningless statement, since extensions *qua* objects have no arguments and, consequently, cannot assume any values (1990, 39). Applying the reasoning from the last paragraph we can see that the examples are not good because they do not fully appreciate the consequences of Frege's view on singular-termhood. Following strictly Frege's explanation in "*Über Begriff und*

Gegenstand”, what (*S*) ends up expressing (due to a logical imperfection of language) is a first-order relation between two objects, namely, the object referred to by ‘the concept $x = x$ ’ and the object referred to by ‘the concept $(\forall x)\varphi(x)$ ’. This relation is not what is originally meant, but it is what we refer to by ‘falls under’ in this particular context. *Q* can be analyzed along the same lines.

¹⁵ My suggestion actually imitates Frege’s explanation of the definite-description operator (λ) in *GGA* §11. The function λx has *a* as value if it takes as argument the extension of a concept under which only *a* falls. But for any other object *b* that is not of this form as argument, the function has *b* itself as value. A consequence of this is that $\lambda \lambda \dots \lambda b = \lambda b = b$ whenever *b* is not an object of the specified form. Although the iteration of λ may be redundant in some contexts, it does not follow that λ is semantically vacuous.

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Department of Philosophy
Federal University
Lgo. Sao Francisco de Paula 1
Rio De Janeiro, R.J.
20051 070
Brazil